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CAPILLARY PRESSURE AND THE GRAVITY AND DYNAMIC PHASE DISTRIBUTION IN A WATER–OIL–GAS–ROCK SYSTEM

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The concept of a four-phase water–oil–gas–rock system as the aggregate of four continua that interpenetrate but do not mix, in a physicochemical sense, with one another assumes that at each "point" of the space occupied by the system the pressures undergo jumps at the interfaces.

Let s_0, s, and s_1 be the saturations of the pore space with gas, oil, and water, respectively ($s_0+s+s_1 = 1$). In equilibrium between the pressures p_0, p, and p_1 in the phases two constitutive relations, $p = p_0 + F(s, s_1)$ and $p_1 = p_0 - F_1(s, s_1)$, should hold. Here the functions F and F_1 are assumed to be specified in the inner region of a triangular diagram (Fig. 1). They depend at least on the structure of the pore space, the physicochemical properties of liquids and rock, which are characterized by a set of certain parameters, and on the saturations.

The problem of specifying constitutive relations is rather complex and poorly studied both experimentally and theoretically. However, the situation is simplified if one of the variable phases vanishes, which corresponds to the boundaries of the triangular diagram. Based on experimental data for a wide range of porous media saturated with various combinations of the pairs of liquids of the electrolyte solution (or water)–kerosene–air type, Leverett [1] has established for these limiting cases that

$$s = 0: \qquad p_1 = p_0 - p_c^1\varphi(s_0) = p_0 - p_c^1\varphi(1 - s_1); \tag{1}$$

$$s_1 = 0: \qquad p = p_0 - p_c^0\varphi(s_0) = p_0 - p_c^0\varphi(1 - s); \tag{2}$$

$$s_0 = 0: \qquad p = p_1 + p_c^{01}\varphi(s) = p_1 + p_c^{01}\varphi(1 - s_1). \tag{3}$$

The parameters p_c^0, p_c^1, and p_c^{01}, which are usually determined in equilibrium, indicate the degree of wettability of the medium skeleton by corresponding liquids and are proportional to the paired differences of specific cohesion and adhesion works of the particles of these liquids. Ignoring the work of cohesion (adhesion) forces of the gas particles, one can assume that approximately $p_c^{01} = p_c^1 - p_c^0$. For $p_c^{01} > 0$, a porous medium is considered hydrophilic, and for $p_c^{01} < 0$, after an appropriate replacement of the variable s for s_1 this medium is considered hydrophobic. However, it should be noted that in changing the direction of the motion of liquids the sign of p_c^{01} can change [2].

The monotonically decreasing function $\varphi(s)$, which coincides up to a factor with the known Leverett function, characterizes the pore-space geometry and is determined by the parameters of the size distribution of conventional pore radii.

We assume that saturations are normalized so that $\varphi(0) = 0$ and $\varphi(1) = 1$. In this case, the parameters p_c^0, p_c^1, and p_c^{01} are simultaneously the maximum possible capillary pressures.

Constitutive Relations. Let at a certain s fixed at the left boundary of the diagram, where $s_0 = 0$ and $s_1 = 1 - s$ and formula (3) is valid, a shift occur inside the triangular region toward the right boundary at which $s_1 = 0$ and $s_0 = 1 - s$. This shift decreases the water saturation s_1 and increases the curvature of water menisci, thus leading to a larger capillary jump at the oil–water interface. On the one hand, this jump can be approximately estimated by the same relation (3) with the oil saturation s increased by the

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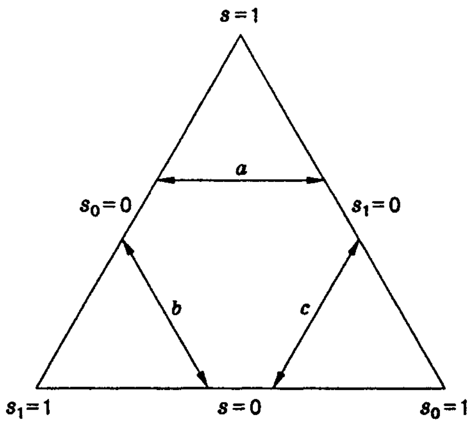

Fig. 1

increase of the gas saturation s_0 after the shift. In other words, since inside the region $1-s_1=s+s_0$, we set $p=p^1=p_1+p_c^{01}\varphi(1-s_1)=p_1+p_c^{01}\varphi(s+s_0)$.

On the other hand, under small shifts from the right boundary of the diagram to the left one, formula (2) allows one to write the following approximation: $p=p^2=p_0-p_c^0\varphi(1-s)=p_0\varphi(s_0+s_1)$.

Considering an arbitrary internal point of the diagram as a result of the displacement from both boundaries (we call these displacements the a transitions), to calculate the oil pressure p, we use the weighted average value of $p=p^1 s_1/(s_1+s_0)+p^2 s_0/(s_1+s_0)$, or in expanded form

$$p=[p_1+p_c^{01}\varphi(s+s_0)]\frac{s_1}{s_1+s_0}+[p_0-p_c^0\varphi(s_0+s_1)]\frac{s_0}{s_1+s_0}. \tag{4}$$

It is clear that at the above boundaries of the diagram formula (4) transforms into "exact" formulas (3) and (2), respectively.

Similar arguments for b transitions from the left boundary to the base and vice versa lead to

$$p_1=[p_0-p_c^1\varphi(s+s_0)]\frac{s_0}{s+s_0}+[p-p_c^{01}\varphi(s+s_0)]\frac{s}{s+s_0}. \tag{5}$$

The c transitions between the right boundary and the base give rise to the relation

$$p_0=[p_1+p_c^1\varphi(s_0+s)]\frac{s_1}{s_1+s}+[p+p_c^0\varphi(s_0+s_1)]\frac{s}{s_1+s},$$

which does not contradict formulas (4) and (5) and, as one can easily check, is a consequence of them.

Solving Eqs. (4) and (5) in terms of p and p_1, after transformations we write the constitutive relations as

$$\begin{aligned} p&=p_0-p_c^0(1-s_1)\varphi(1-s)-(p_c^1-p_c^{01})s_1\varphi(1-s_1),\\ p_1&=p_0-p_c^0 s\varphi(1-s)-[(1-s)p_c^1+sp_c^{01}]\varphi(1-s_1). \end{aligned} \tag{6}$$

Let us consider the behavior of the obtained relations in the vicinity of particular points-vertices of the triangular diagram, which correspond to the limiting saturations in any phase:

(1) On the $s_0=0$ side of the triangle ($s_1=1-s$), from formulas (6) we formally obtain the relations

$$\begin{aligned} p&=p_0-p_c^0 s\varphi(1-s)-(p_c^1-p_c^{01})(1-s)\varphi(s),\\ p_1&=p^0-p_c^0 s\varphi(1-s)-[(1-s)p_c^1-sp_c^{01}]\varphi(s), \end{aligned}$$

from which the "exact" formula (3) follows due to the exclusion of pressure p_0 in the gas phase that has disappeared. In addition, since $s\to0$, from these expressions we find that $p\to p_0$ and $p_1\to p_0$; therefore $p\to p_1$, and for $s_1\to0$, we have $p\to p_0$ and $p_1\to p_0-p_c^{01}$, i.e., $p-p_1=p_c^{01}$.

(2) For $s \to 0$ ($s_1 \to 1 - s$), first of all, formula (1) follows from relations (6), and furthermore for pressure p in the "disappearing" oil phase, we obtain the expression $p = p_0 - p_c^0 s_0 - (p_c^1 - p_c^{01})(1 - s_0)\varphi(s_0)$, from which follows $p \to p_0$ for $s_0 \to 0$ and $p \to p_0 - p_c^0$ for $s_1 \to 0$;

(3) For $s_1 \to 0$ ($s \to 1 - s_0$), the "exact" formula (2) follows from the first formula of relations (6), and for pressure p_1 in the "disappearing" water phase, we find $p_1 = p_0 - p_c^0(1 - s_0)\varphi(s_0) - s_0(p_c^1 - p_c^{01}) - p_c^{01}$, from which $p_1 \to p_0 - p_c^1$ for $s \to 0$ and $p_1 \to p_0 - p_c^{01}$ for $s_0 \to 0$.

The results of the transitions considered seem reasonable and physically consistent.

Gravity Phase Distribution. As a first trivial example of application of relations (6), we consider a steady gravity phase distribution in the upright direction either in an oil–gas field surrounded by water or in a separate hydrocarbon inclusion in the aeration zone ($s_0 \geqslant 0$) of grounds polluted by petroleum products. Similar problems were considered in [3–5], in which one can find the results of laboratory experiments and some calculations performed using simplified mathematical models. In particular, it was assumed in [3] that the water saturation s_1 depends only on the capillary pressure between oil and water, while the total saturation by liquid phases ($s + s_1$) is a function of the capillary pressure at the oil–gas interface.

Let the y axis be directed vertically upward, its origin $y = 0$ correspond to the positions of the lower boundaries of a hydrocarbon inclusion (deposit) or the aeration zone of grounds, on which $s_0 = 0$ and $s_1 = 1$, and the water pressure $p_1 = 0$. Since the gas (air) density is relatively low, one can assume that its pressure is independent of height, i.e., $p_0 \equiv \text{const}$. In a quiescent state, the liquid pressures should be hydrostatic:

$$p_1 = -\gamma_1 y, \quad p = p^0 - \gamma y.$$

Here γ_1 and γ are the specific weights of the corresponding liquids and p^0 is the oil pressure at height $y = 0$.

For definiteness, we assume that at $y = 0$ the pressures in all phases are equal to zero ($p^0 = p_0 = 0$). Substituting the values of p_0, p, and p_1 in formulas (6), we obtain the system of nonlinear algebraic equations for each fixed y, from which the values of the saturations with water s_1 and oil s are found:

$$s_1 = 1 - \varphi^{-1}\left[\frac{y - h_c^0 s\varphi(1 - s)}{(1 - s)h_c^1 + sh_c^{01}}\right], \quad s = 1 - \varphi^{-1}\left[\frac{\delta y - (h_c^1 - h_c^{01})s_1\varphi(1 - s_1)}{h_c^0(1 - s_1)}\right]. \tag{7}$$

Here $h_c^i = p_c^i/\gamma_1$ ($i = 0$, 1, and 01), $\delta = \gamma/\gamma_1$, and φ^{-1} is a function inverse to the function φ.

Figure 2 shows the plots of the solutions of system (7) of the gravity phase distribution for the function of capillary pressure $\varphi(s) = (0.1s/(1.1 - s))^{1/2}$ and $h_c^0 = 15$. Curves 1–3 correspond to $h_c^1 = 20$ and $h_c^{01} = 5$, $h_c^1 = 20$ and $h_c^{01} = 1$, and $h_c^1 = 16$ and $h_c^{01} = 1$, respectively.

Evidently, the relation of capillary pressures h_c^0, h_c^1, and h_c^{01} affect significantly the character of the phase distribution. In particular, for immiscible liquids which wet the rock skeleton almost identically, a gas-free zone is formed in the lower part of the inclusion (curves 2 and 3).

Internal Capillary Locking of Two Phases upon Quasi-Three-Phase Filtration. As the second example of application of relations (6), which close the system of equations of motion and also of the laws of mass conservation under three-phase filtration, we consider the problem of limiting equilibrium of an oil–gas inclusion with air flow passing through this inclusion. As in the case of quasi-two-phase filtration [6], we assume that the gas and oil are in an immovable capillary-locked state.

In this case, from the equations of motion it follows that the pressures (heads) in immovable phases are constant ($h \equiv h^0$ and $h_0 \equiv h_0^0$). Confining ourselves to a one-phase motion, we select the direction of the x axis opposite to the velocity vector of the incoming water flow and its origin $x = 0$ corresponding to the outlet section of the inclusion located in a certain region $0 \leqslant x \leqslant a$.

In this region, water filtration obeys the generalized Darcy's law and the continuity equation

$$v_1 = -k_1 f_1(s_1)\frac{\partial h_1}{\partial x}, \quad \frac{\partial v_1}{\partial x} = 0. \tag{8}$$

Let there be a uniform filtration water flow beyond the inclusion. This flow moves under the action of the hydraulic gradient i_0. From the second equation of system (8) and the condition of conjugate flows we have $v_1 = -i_0 k_1$.

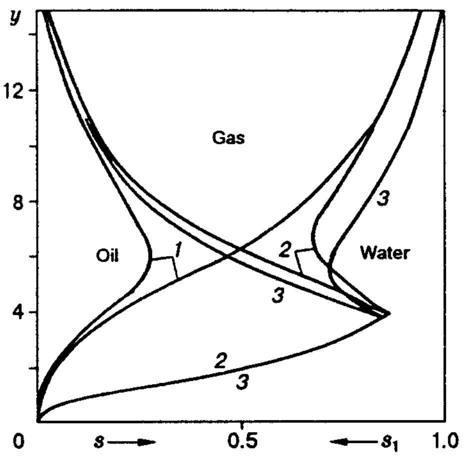

Fig. 2

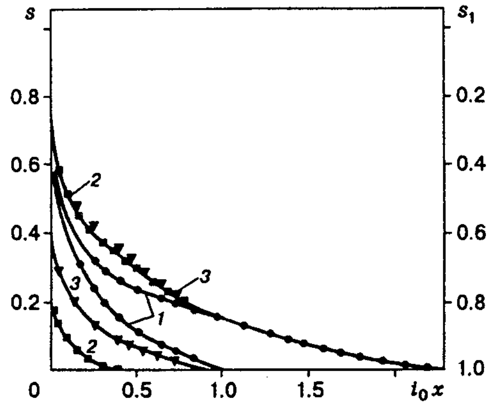

Fig. 3

By the limiting equilibrium we mean a state of the hydrocarbon inclusion in which the water saturation equals zero ($s_1 = 0$) in the outlet section. An increase in the hydraulic gradient of the incoming filtration flow leads to fracture of the inclusion which has been previously in a state of limiting equilibrium [6].

It should be noted that at the point $x = 0$ of water discharge from the oil-gas inclusion, none of the capillary-locked phases can reach the limiting values $s(0) = 1$ or $s(0) = 0$ [$s_0(0) = 1$], since, as is readily seen, these values correspond to the cases of capillary locking of either an oil or gas inclusion alone. This imposes a certain restriction on the assignment of the difference between the constants $h^0 - h^0_0$. Choosing the reference point for heads such that the gas pressure is equal to zero, $h^0_0 = 0$ ($p_0 \equiv 0$), we see that this restriction is reduced to the inequality $h^0 \geqslant -h^0_c$.

Let us introduce a dimensionless parameter d ($0 \leqslant d \leqslant 1$), assuming that $h^0 = -dh^0_c$. Then, having defined the function $z(s_1) = [d - d_c s_1 \varphi(1 - s_1)]/(1 - s_1)$, where $d_c = (h^1_c - h^{01}_c)/h^0_c$ from the first equation of relations (6), we find

$$s(s_1) = 1 - \varphi^{-1}(z(s_1)). \tag{9}$$

Thus, in agreement with formula (9), the oil saturation at the outlet $s(0)$ is completely determined by a given parameter d, since $s(0) = 1 - \varphi^{-1}(d)$. The limiting values of this parameter ($d = 0$ and 1) correspond to the degenerated case of locking of one of the phases.

For $d_c \leqslant 1$ and $d \in (0, 1)$, a value of water saturation $s_1 = s^0_1$ can be found at which $z(s^0_1) = 1$ [$s(s^0_1) = 0$]. Let this value be reached at the point $x = x^0$ of the inclusion. This implies that for $x > x^0$, only the gas can be in a capillary-locked state and $s \equiv 0$. From the monotonic character of the function φ, it follows that s^0_1 is uniquely found in the above range of variation of the parameters d and d_c as a root of the transcendental equation $s^0_1 = (1 - d)/(1 - d_c\varphi(1 - s^0_1))$. The saturation distributions $s(x)$, $s_1(x)$, and $s_0(x) = 1 - (s + s_1)$ are obtained by integrating Eq. (8) with allowance for relations (6) and (9) and the boundary condition $s_1(0) = 0$ in the form

$$i_0 x = I(s_1) - f_1(s_1)g(s_1, s), \tag{10}$$

where

$$g(s_1, s) = h^0_c s z(s_1) + [(1 - s)h^1_c + s h^{01}_c]\varphi(1 - s_1); \quad I(s_1) = \int\limits_0^{s_1} g(s_1, s) f_1'(s_1)\, ds_1.$$

For $s^0_1 < s_1 < 1$ ($x^0 < x \leqslant a$), one should set in expression (10) $s \equiv 0$ and $g(s_1, 0) = g_0(s_1) =$

$h_c^1\varphi(1 - s_1)$. In particular, for the maximum size a of the oil–gas inclusion, we obtain the formula

$$a = \Big[I(s_1^0) + \int_{s_1^0}^{1} g_0(s_1)f_1' ds_1\Big]/i_0.$$

Figure 3 shows the diagrams of the saturation distribution in an oil–gas inclusion with internal capillary locking of the oil and gas phases at limiting equilibrium with water flow moving under the action of the head gradient at infinity i_0. Curves 1 correspond to $h_c^0 = 10$, $h_c^1 = 20$, $h_c^{01} = 10$, and $d = 0.3$; 2 to $h_c^0 = 10$, $h_c^1 = 20$, $h_c^{01} = 10$, and $d = 0.5$; and 3 to $h_c^0 = 15$, $h_c^1 = 20$, $h_c^{01} = 15$, and $d = 0.3$. In calculations, the relative phase permeability f_1 for the water phase is used as a power function of $f_1(s_1) = s_1^{3.5}$.

In conclusion, we shall formulate the basic results:

(1) Using a linear-fractional interpolation from the boundaries inside the triangular phase diagram, we have obtained relations between the pressure jumps in the phases and the saturations for a system of the water–oil–gas–rock type. To do this, one should know the Leverett function that characterizes the pore space of the medium and the characteristic capillary pressures for each pair of fluids saturating the medium. These are the constitutive relations in equations describing the three-phase filtration process with allowance for the action of capillary forces.

(2) Based on the relations derived, we have solved two model problems: the problem of static equilibrium of phases in the gravity field under pollution of the aeration zone of a water-bearing horizon by petroleum products and also the problem of internal capillary locking of the oil–gas inclusion in uniform filtration water flow.

(3) As multivariant calculations show, the phase-distribution pattern in the region of inclusions depends significantly on the relation of the characteristic values of capillary pressures in the two problems.

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